Hermann Minkowski
Founder of geometry of numbers

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Minkowski put forward his concept of space-time, or Minkowski space as it is sometime called, in 1907 in his book *Space and Time*. Einstein himself was very forthright about the extent to which the theory of relativity depended on Minkowski's innovatory work. Space-time was a useful and elegant format for special relativity, and was essential for general relativity, published in 1916, in which space-time is allowed to be curved. It is the curvature of space-time that accounted for the phenomenon of gravitation.

A Dictionary of Scientists, Oxford University Press, 1999

"The views of space and time I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

Hermann Minkowski

Hermann Minkowski's contributions to the development of modern mathematics are very significant. His created the basis for modern functional analysis. He extended the knowledge of quadratic forms to a great extent. He founded a sub-discipline of mathematics called geometry of numbers.

It was Mnkowski who laid the mathematical foundation for Albert Einstein's theory of relativity. Minkowski could realise that Einstein's special theory of relativity could be best appreciated in a non-Euclidean space (now called Minkowski space). Thus while developing the rigorous mathematical structure and geometrical implications of the theory of relativity Minkowski proposed space and time, which were earlier thought to be independent, were linked together in a four-dimensional 'space-time continuum'. The space-time continuum, also called space-time, was conceived by Minkowski to denote the geometry of the physical universe as suggested by the theory of relativity. As we know, in Newtonian physics or classical physics space and time were considered quite separate quantities. But Minkowski demonstrated that the concept of relativity theory made it necessary mathematically to take time into account as a fourth dimension in addition to the three spatial dimension, viz., length, breadth and width. Thus the works of Einstein and Minkowski showed that space and time are actually intimately interlinked. The totality of a space and time as a single four-dimensional continuum is also referred to as event universe or Minkowski

Hermann Minkowski

universe in which the history of a single space point in the course of time must be treated as a curve or line and an event limited both in space and time represents a point. These geometric concepts in Minkowski's universe are often referred as world curves or world lines and world points respectively are to be distinguished from their analogs in normal three-dimensional space. It should be mentioned here that the origin of the idea of the unity of space and time could be traced to the analysis of transformations by Hendrik Lorentz. Minkowski for the first time realised the importance of the Lorentz transformation ideas.

Before Minkowski came up with his space-time model, Einstein's special theory of relativity was viewed by physicists including Einstein himself as a physical theory without realising its geometrical implications. It was Minkowski who studied the mathematical structure of the new theory and its geometrical implications. Minkowski's mathematical ideas and techniques played an instrumental role in Einstein's construction of the general theory of relativity.

Both Minkowski and David Hilbert influenced each other's career. They first came in contact as students in Konigsberg. Minkowski was junior to Hilbert. Later they became colleagues at the University of Gottingen. Both of them moved from pure mathematics to mathematical physics. They were influenced by and reinforced the idea of a "pre-established harmony between mathematics and physics." It was also believed that mathematical sophistication was essential in unveiling the secret of nature. It is well-known that Hilbert greatly influenced the course of mathematical research in the 20th century by suggesting a list of major mathematical problems in his famous 1900 lecture. However, it was not very well-known that it was Minkowski who suggested to Hilbert to take up this theme for his lecture. Minkowski in a letter to Hilbert wrote: "What would have the greatest impact would be an attempt to give a preview of the future, i.e., a sketch of the problems with which future mathematicians should occupy themselves. In this way you could perhaps make sure that people would talk about your lecture for decades in the future." Minkowski and Hilbert were greatly responsible for changing Einstein's perception of mathematics from 'a mere tool

in the service of physical intuition' to 'as the very source of scientific creativity.'

Minkowski taught Albert Einstein mathematics at the Zurich Polytechnic. He had no particular fascination for Einstein as a student. In his correspondence with Hilbert the only student of the Polytechnic referred to was Walter Ritz (1878-1909). He is believed to have said to his later students that he found Einstein's presentation of theory of relativity mathematically awkward.

Walter Ritz

Wilhelm Wien

Ernst Eduard Kummer

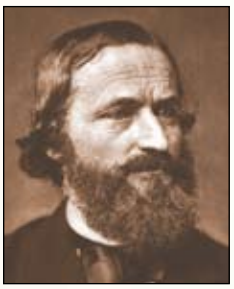
Leopold Kronecker

Hermann von Helmholtz

Hermann Minkowski was born on 22 June 1864 at Alexoten (Alexotas) in the Russian empire under Tsar Alexander II (near Kaunas in modern-day Lithuania). His parents Lewin Minkowski and Rachel Minkowski (*nee* Taubmann) were of German origin. Minkowski was taught at home till the age of seven. In 1872, his parents moved back to Germany and settled in Konigsberg (now Kalinigrad, Russia). Minkowski attended the Altstadtisches Gymnasium where among his juniors were Wilhelm Wien (1864-1928) and Arnold Sommerfeld (1868-1951), who later became distinguished physicists.

Minkowski joined the University of Konigsberg in April 1880, where he was taught by among others Heinrich Weber, Woldemar Voigt, Adolf Hurwitz and Ferdinand Lindermann. He spent three terms in the University of Berlin where he attended lectures of Ernst Eduard Kummer (1810-1893), Leopold Kronecker (1823-1891), Hermann von Helmholtz (1821-1894), and Gustav Robert Kirchhoff (1824-1887).

In 1883, Minkowski won the Grand Prix des Sciences Mathematiques of the Paris Academy of Sciences. Minkowski was only 18 years old. He shared the award with the accomplished British mathematician Henry J. S. Smith. The prize was announced in 1881. The topic of the prize was finding a solution to the problem of the number of representations of an integer as the sum of five squares. The problem was earlier tackled by Eisenstein in 1847 and he gave a formula for the number of such representation. However, he had not given any explanation on how he arrived at the formula. In 1867, Smith solved the problem and he also gave proof in support of his result. The Paris Academy of Sciences was not aware of Smith's work while announcing the topic of the prize. Smith submitted an elaborate version of his earlier work on the topic. Minkowski also came up with a solution to the problem while reconstructing Eisenstein's theory of quadratic forms. Minkowski submitted his results to the Academy in the form of a 140-page manuscript. Minkowski's formulation was considered better than that of Minkowski as the former used more natural and general definitions in working out the proof. In 1885, he completed his PhD at Konigsberg under the supervision of Lindermann. His PhD thesis was on quadratic forms. At Konigsberg University Minkwoski came in contact David Hilbert.



Gustav Robert Kirchhoff

Henry J. S. Smith

After PhD he had to undertake obligatory military service and then in 1887 he joined the Bonn University, as a Privatdozent (non-salaried lecturer). In 1892 he was promoted to the rank of Associate Professor. At Bonn started working in the field of mathematical physics. The first problem to be undertaken by him was to find the motion of solids immersed in perfect liquid. This problem was earlier studied by W. Thomson, Kirchhoff, Clebsch, and others. Minkowski developed a method which could be applied to a solid irrespective of its form. At Bonn, Minkowski's interest moved from mathematics to physics. He started spending time at the Bonn's Institute of Physics which at the time was headed by Heinrich Hertz. He even attended a laboratory course. There are reports to indicate that Hertz invited young Minkowski over for dinner. Minkowski's interaction with Hertz did not last long as the latter died in 1894. In the same year Minkowski left Bonn. Perhaps things could have been different if Hertz had lived longer. But it had some decisive impact on Minkowski, he developed an interest in theoretical mechanics. It may be noted that Hertz devised new principles of mechanics in the early 1890s.

While at Bonn, Minkowski also worked on number theory, which led to the formation of the basis of a new sub-discipline in mathematics, the geometry of numbers. In 1896, he gave a detailed account of his geometry of numbers in which he developed geometrical methods for the treatment of certain problems of number theory. He discussed his results in his book on geometry of numbers, Geometrie der Zahlen.

In 1894, Minkowski joined the faculty of Zurich Polytechnic, where he joined his former teacher Hurwitz. At Zurich, Minkowski was given a much higher salary and he also got the opportunity to interact with students of engineering and mathematics. He spent about six years at the polytechnic and gave lectures on variety of topics namely analytical mechanics, hydrodynamics, potential theory, variational calculus, number theory, the theory of functions, partial differential equations and algebra. While teaching at the Zurich Polytechnic, Minkowski wanted to teach at Zurich University but he was not permitted to do so. He was not very happy with the situation as he considered the Polytechnic as a school "from which a complete knowledge of mathematics could not be obtained."

In 1902, Minkowski returned to the University system, this time Gottingen University. A new chair (this was to be the third chair) of mathematics was specifically created for Minkowski. It was an extraordinary move. This was possible because of interest taken by Hilbert in inviting Minkowski to Gottingen. Hilbert had earlier moved to Gottingen at the invitation of Felix Klein (1849-1925), the German mathematician, one of the great formative influences on the development of modern geometry. Klein had taken up the chair of mathematics at the Gottingen University in 1886 and took up the task of building Gottingen into a great centre for mathematics. To realise his objective Klein persuaded the authorities to create another chair in pure mathematics and invited Hilbert to occupy it. Hilbert had already an offer from the Berlin University to join the Lazarus Fuchs' chair in mathematics. Hilbert decided to take up the Gottingen offer provided Minkowski was allowed to join him and Klein at the Gottingen University. It was at Klein's insistence that the Prussian educational authorities decided to create an unprecedented third chair of mathematics so that both Hilbert and Minkowski could be brought to Gottingen. After coming to Gottingen Minkowski was mainly concerned with mathematical physics. In 1905 Minkowski joined Hilbert in organizing a seminar for reviewing the progress in the theories of electron. In 1907 they conducted a joint seminar on the equations of electrodynamics. In the remaining two years of his life (1907-1909) Minkowski was totally involved in the study of the equations of electrodynamics and the theory of relativity postulated by Einstein. Minkowski reformulated the special theory of relativity in terms of space-time continuum. He demonstrated that the main conclusions of the theory could be derived by using only mathematical principles and there was no need to take recourse to experiments. Minkowski's treatment raised the theory to 'a level of clarity and sophistication that surpassed by far Einstein's original one.' Well-known physicists like Max von Laue and Arnold Sommerfeld further extended Minkowski's ideas. Laue's introductory textbook on the special theory of relativity, published in 1911, happened to be the first textbook on the subject that used Minowski's formulation.

Minkowski wanted to settle the Four-colour Map conjecture, but eventually he did not succeed. It is interesting to note that before he took up this problem he had remarked that the problem was not solved because only third-rate mathematicians worked on it and he had also asserted: "I believe I can prove it." But later on realising that he could not provide a satisfactory proof he said: "Heaven is angered by my arrogance, my proof is also defective."

Minkowski died on 12 January 1909 in Gottingen, Germany. His death was sudden. He was just 44 years old at the time of his death.

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6. Available literature on the Internet.

(This article is a popular presentation of important points on the life and work of Hermann Minkowski available in the existing literature. The idea is to inspire younger generation to know more about Minkowski. The author has given the sources consulted for writing this article. However, the sources on the Internet are numerous and so they have not been individually listed. The author is grateful to all those authors whose works have contributed to writing this article.)

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